



LOOKING FURTHER

Roadmap

• Warm up

- Check for normality
- Main statistical tests
- p-value corrections
- Effect size



Context and assumptions

• We focus on quantitative **variables** only

- \bigcirc nominal
- \bigcirc ordinal
- 🔿 interval
- 🔿 ratio

the

- Factors are nominal or ordinal
- Dependent variables are ratios

Our statistical tests detect differences between the **means** of dependent variable

• Treatments are **<u>fixed</u>** a priori



Tasks for data analysis

1. Descriptive statistics

○ for understanding the "shape" of collected data

2. Select statistical test

- $\,\odot\,$ according to collected metrics and data distribution
- \bigcirc this might involve also data transformation

3. Hypothesis testing

- \bigcirc for providing evidence about your findings
 - i. statistical significance

4. Effect size calculation

 for understanding if your (statistically significant) results are actually relevant in practice



What is a statistical test?

- Calculation of a sample statistic assuming that the null hypothesis is true
- The calculated value of the statistic has a certain probability given that the null hypothesis is true (pvalue)

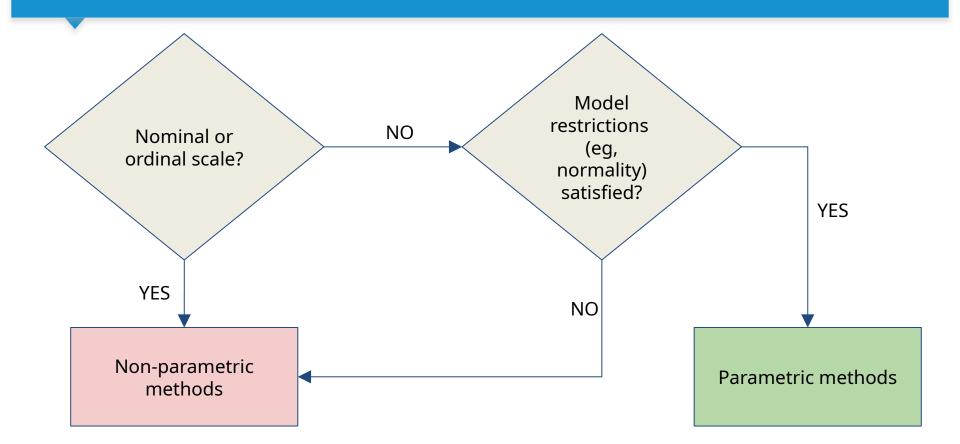


First choice: parametric VS non-parametric tests

- Parametric tests assume a specific distribution of the data
 - typically, normal distribution
 - more powerful
 - → lower chances of having Type II errors
- Non-parametric tests do not make any assumption about data distribution
 - \bigcirc more general
 - less powerful
 - \rightarrow larger samples are needed



How to choose?



If you can apply it, always prefer a parametric test

In your report explain in details why you choose a

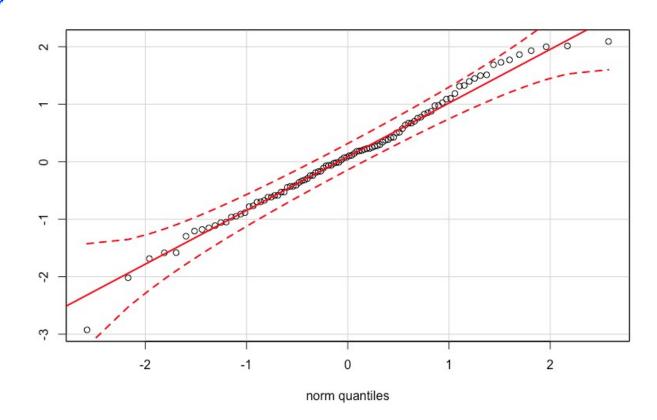
7 Ivano Malavolta / S2 group / Statistical tesspecificztest!

Check for normality



Graphical check (Q-Q plot)

- > y <- rnorm(100)
- > library(car)
- > qqPlot(y)





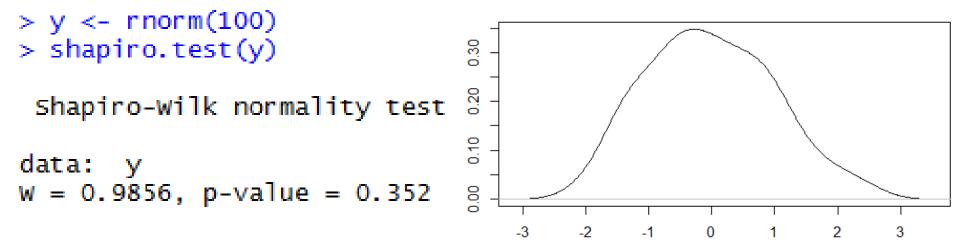
Normality tests

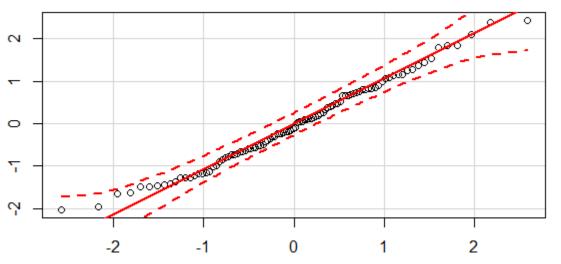
- Normality tests
 - \bigcirc H₀: sample is drawn from a normal distribution
- Shapiro-Wilk test (AKA Shapiro-Wilk's W)

 If p-value <α for a given sample, we can conclude data is <u>NOT</u> normally distributed



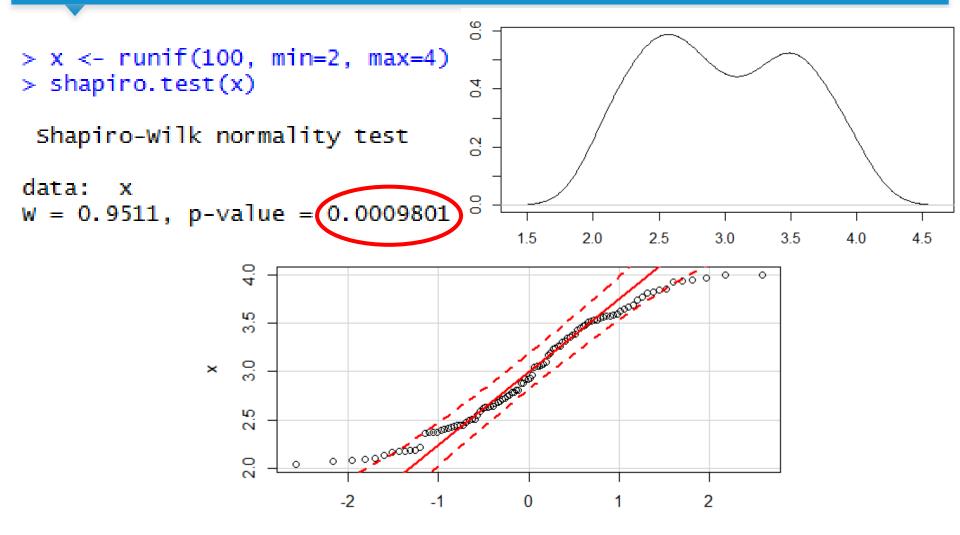
Shapiro-Wilk test







Shapiro-Wilk test





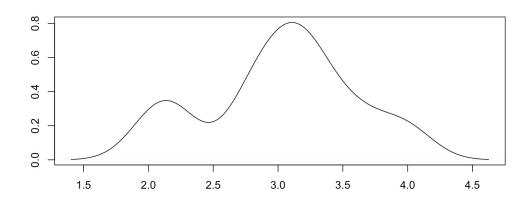
Shapiro-Wilk test

Warning: Shapiro-Wilk is **not** robust for small samples!

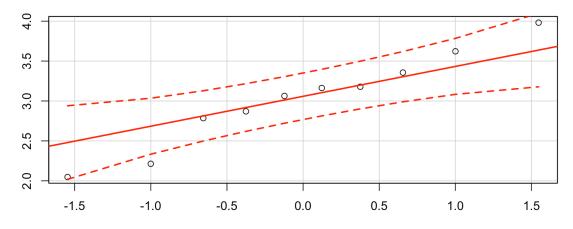
○ Additional verification (e.g. via Q-Q plot) is always needed

- > x <- runif(10, min=2, max=4)
 > qqPlot(x)
- > shapiro.test(x)

Shapiro-Wilk normality test



data: x
W = 0.96708, p-value = 0.8625





Inspiration for checking assumptions

Check the papers EASE_2020 and MobileSoft_2020 on Canvas

A nice online resource is also available here: <u>https://www.datanovia.com/en/lessons/t-test-in-r/#assump</u> <u>tions-and-preliminary-tests-1</u>



Main statistical tests



Statistical tests VS experiment design

Parametric	Non-parametric
	Chi-2, Binomial test
t-test, F-test	Mann-Whitney, Chi-2
Paired t-test	Wilcoxon, Sign test
ANOVA	Kruskal-Wallis, Chi-2
ANOVA ^a	
	t-test, F-test Paired t-test ANOVA



One factor - 2 treatments - random design

Parametric	Non-parametric
	Chi-2, Binomial test
t-test, F-test	Mann-Whitney, Chi-2
Paired t-test ANOVA	Wilcoxon, Sign test Kruskal-Wallis, Chi-2
ANOVA ^a	
	t-test, F-test Paired t-test ANOVA





Goal: compare <u>independent</u> samples

Parametric

- > Values of the dependent variable obtained with different treatments
- For each treatment you are measuring different subjects

Hypotheses:

- Two-tailed \bigcirc H₀: $\mu_2 = \mu_1$ H_a: $\mu_2 \neq \mu_1$
- One-tailed (alternative: greater) \bigcirc H₀: $\mu_2 = \mu_1$ H_a: $\mu_2 > \mu_1$
 - One-tailed (alternative: less) \bigcirc H₀: $\mu_2 = \mu_1$ H_a: $\mu_2 < \mu_1$

More powerful

Cannot say anything in the opposite direction



t-Test in R

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```
## Default S3 method:
                t.test(x, y = NULL)
                        alternative = c("two.sided", "less", "greater"),
                        mu = 0, paired = FALSE, var.equal = FALSE,
                        conf.level = 0.95, ...)
                ## S3 method for class 'formula'
                t.test(formula, data, subset, na.action, ...)
                Arguments
                х
                              a (non-empty) numeric vector of data values.
                У
                              an optional (non-empty) numeric vector of data values.
                 alternative
                              a character string specifying the alternative hypothesis, must be one of
                              "two.sided" (default), "greater" or "less". You can specify just the initial
                              letter.
                mu
                              a number indicating the true value of the mean (or difference in means if you
                              are performing a two sample test).
                paired
                              a logical indicating whether you want a paired t-test.
                var.equal
                              a logical variable indicating whether to treat the two variances as being equal.
                              If TRUE then the pooled variance is used to estimate the variance otherwise the
Ivano Malavoli
                              Welch (or Satterthwaite) approximation to the degrees of freedom is used.
```

VU

t-Test: example

```
> x <- rnorm(100)
> y <- rnorm(100)
> t.test(x,y)
Welch Two Sample t-test
data: x and y
t = -0.6148, df = 196.807, p-value = 0.5394
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
-0.3533052 0.1853781
sample estimates:
    mean of x mean of y
-0.03704463 0.04691890
```



t-Test: example 2

```
> x <- rnorm(100)
> y <- rnorm(100, mean=5)
> t.test(x,y)
```

Welch Two Sample t-test

```
data: x and y
t = -35.219, df = 197.704, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
   -5.217552 -4.664236
sample estimates:
   mean of x mean of y
0.004072809 4.944966734</pre>
```



Mann-Whitney test

Goal: compare independent samples

Nonparametric

It can be used instead of the t-test when data is not normal

Hypotheses:

- Two-tailed \bigcirc H₀: $\mu_2 = \mu_1$ H_a: $\mu_2 \neq \mu_1$
- One-tailed (alternative: greater) \bigcirc H₀: $\mu_2 = \mu_1$ H_a: $\mu_2 > \mu_1$
- One-tailed (alternative: less) \bigcirc H₀: $\mu_2 = \mu_1$ H_a: $\mu_2 < \mu_1$

Same hypotheses as the t-test



Mann-Whitney test in R

```
wilcox.test(x, ...)
## Default S3 method:
wilcox.test(x, y = NULL,
             alternative = c("two.sided", "less", "greater"),
             mu = 0, paired = FALSE, exact = NULL, correct = TRUE,
             conf.int = FALSE, conf.level = 0.95, ...)
## S3 method for class 'formula'
wilcox.test(formula, data, subset, na.action, ...)
Arguments
х
             numeric vector of data values. Non-finite (e.g., infinite or missing) values will be omitted.
У
             an optional numeric vector of data values: as with x non-finite values will be omitted.
alternative
             a character string specifying the alternative hypothesis, must be one of "two.sided" (default),
              "greater" or "less". You can specify just the initial letter.
mu
             a number specifying an optional parameter used to form the null hypothesis. See 'Details'.
paired
             a logical indicating whether you want a paired test.
exact
             a logical indicating whether an exact p-value should be computed.
correct
             a logical indicating whether to apply continuity correction in the normal approximation for the
             p-value.
```

VU

Mann-Whitney test: example

- > x <- runif(100)
 > y <- rexp(100)</pre>
- > wilcox.test(x,y)

Wilcoxon rank sum test with continuity correction

```
data: x and y
W = 3862, p-value = 0.005447
alternative hypothesis: true location shift is not equal to 0
```



One factor - 2 treatments - paired design

Design	Parametric	Non-parametric
One factor, one treatment		Chi-2, Binomial test
One factor, two treatments, completely randomized design	t-test, F-test	Mann-Whitney, Chi-2
	D 1 1	
One factor, two treatments, paired comparison	Paired t-test	Wilcoxon, Sign test
One factor, more than two treatments	ANOVA	Kruskal-Wallis, Chi-2
More than one factor	ANOVA ^a	



Paired t-Test

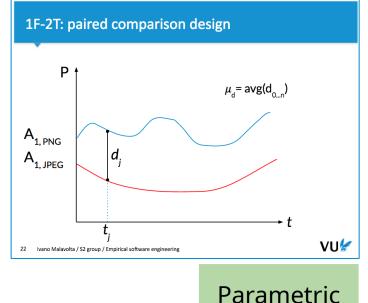
Goal: compare independent samples from repeated measures

- Each subject receives different treatments
- We focus on the differences exhibited by each subject with different

Hypothesets.

Samples must be equal in size

- Two-tailed \bigcirc H₀: $\mu_d = 0$ H_a: $\mu_d \neq 0$
- One-tailed (alternative: greater) \bigcirc H₀: $\mu_d = 0$ H_a: $\mu_d > 0$
- One-tailed (alternative: less) \bigcirc H₀: $\mu_d = 0$ H_a: $\mu_d < 0$





Paired t-Test: example

```
> x <- rnorm(100)
> y <- rnorm(100, mean=5)
> t.test(x,y, paired=TRUE)
Paired t-test
data: x and y
t = -34.0292, df = 99, p-value 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
-5.364349 -4.773235
sample estimates:
mean of the differences
-5.068792
```



Wilcoxon signed-rank test

Goal: compare independent samples from repeated measures

Hypotheses.



- One-tailed (alternative: greater) \bigcirc H₀: $\mu_d = 0$ H_a: $\mu_d > 0$

paired t-test

Same hypotheses as the

Non-

parametric

• One-tailed (alternative: less) $\bigcirc H_0: \mu_d = 0$ $H_a: \mu_d < 0$

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Wilcoxon signed-rank test: example

> x <- runif(100)
> y <- rexp(100)
> wilcox.test(x,y, paired=TRUE)

Wilcoxon signed rank test with continuity correction

data: x and y v = 1110, p-value = 1.153e-06 alternative hypothesis: true location shift is not equal to 0



>=1 factors - >2 treatments

Design	Parametric	Non-parametric
One factor, one treatment		Chi-2, Binomial test
One factor, two treatments, completely randomized design	t-test, F-test	Mann-Whitney, Chi-2
One factor, two treatments, paired comparison	Paired t-test	Wilcoxon, Sign test
One factor, more than two treatments	ANOVA	Kruskal-Wallis, Chi-2
More than one factor	ANOVA ^a	



ANOVA (ANalysis Of VAriance)

Goal: understand how much of the total variance is due to differences <u>within</u> factors, and how much is due to differences <u>across</u> factors

- Many types of ANOVA tests
- Works for many experiment designs

Hypotheses:

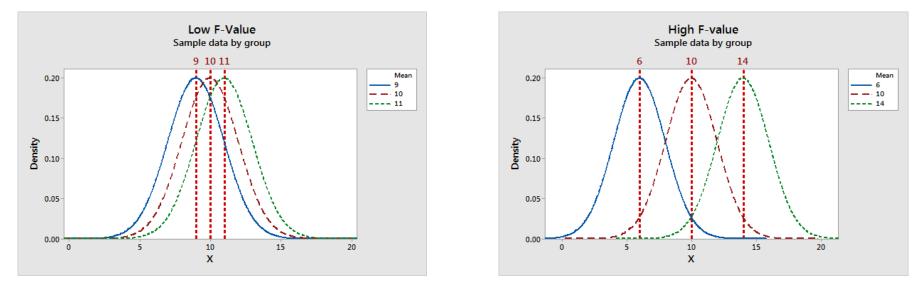
 $H_{0}: \mu_{1} = \mu_{2} = \mu_{3} \qquad H_{a}: \mu_{1} \neq \mu_{2} \vee \mu_{1} \neq \mu_{3} \vee \mu_{2} \neq \mu_{3}$

Parametric



F-statistic

F = Variation <u>among</u> sample means / variation <u>within</u> the samples



when $H_0 \rightarrow F$ follows a known F-distribution

³² https://goo.gl/jHWo8A</sup> to be 1





F tends to be larger if H₀ is false

 \rightarrow the more F deviates from 1, the stronger the evidence for unequal population variances

- Methods to determine significance level:
 - \bigcirc *textbook:* compare F against a table of critical values (according to DF and α). If F > F_{critical}, reject H₀
 - \bigcirc *computer-based:* compute the p-value of finding F greater than the observed value. If p < α , reject H₀



Types of ANOVA

One-way ANOVA

```
○ one factor, >2 treatments
```

○ if 2 treatments: equivalent to *t-test* (almost never

```
      > summary(data$Watts)

      Min. 1st Qu. Median Mean 3rd Qu. Max.

      207.3
      214.0
      214.2
      215.7
      219.8
      222.2

      > summary(data$Case)
      mysql_modified mysql_original mysql_vanilla
      6

      10
      10
      10
```

```
#one-way
```

```
data <- read.csv('practice_1_power.csv')
data.aov <- aov(Watts~Case, data=data)
summary(data.aov)</pre>
```



Types of ANOVA

Factorial ANOVA

- \odot 2 (two-way) or more factors
- \bigcirc any number of treatments

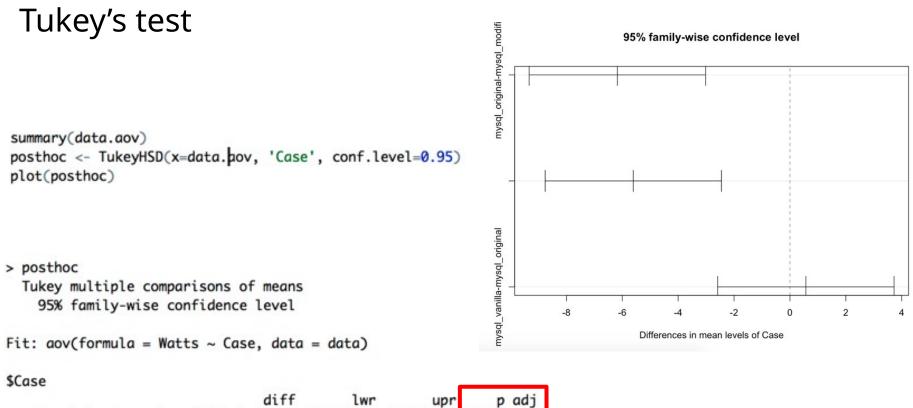
○ also computes interactions

```
# #two-way
server <- factor(sample(1:3, 30, replace=TRUE), levels=c(1:3), labels=c('Server 1', 'Server 2', 'Server 3'))
data_new <- cbind(server, data)
data.2aov <- aov(Watts~Case*server, data=data_new)
summary(data.2aov)
#</pre>
```

> summary(data.2aov) Df Sum Sq Mean Sq F value Pr(>F)2 232.95 116.48 15.421 7.57e-05 *** Case 2 32.22 16.11 2.133 0.143 server Case:server 4 27.80 6.95 0.920 0.471 Residuals 21 158.62 7.55 ----0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 Signif. codes:



How to know which treatments really differ?



	diff	lwr	upr	p adj
<pre>mysql_original-mysql_modified</pre>	-6.176	-9.331299	-3.020701	0.0001302
mysql_vanilla-mysql_modified	-5.605	-8.760299	-2.449701	0.0004301
mysql_vanilla-mysql_original	0.571	-2.584299	3.726299	0.8953945



ANOVA assumptions

The dependent variable should be **continuous**

- Samples must be **independent**
- Normal distribution of the dependent variable between the groups (approximately)
- Residuals (aka errors in the sample) should be normally distributed

O qqPlot(residuals(myData.aov))

Homoscedasticity

Assumptions violated

 \rightarrow non-parametric alternative

variance between groups should be the same

leveneTest(x ~ y, data=myData)

ANOVA: non-parametric alternative

Kruskal-Wallis: one-way non-parametric ANOVA

one factor, multiple treatments ono estimate of the treatment effect (due to ranking)

#non-parametric one-way
kruskal.test(Watts~Case, data=data)

> kruskal.test(Watts~Case, data=data)

Kruskal-Wallis rank sum test

data: Watts by Case
Kruskal-Wallis chi-squared = 12.718, df = 2, p-value = 0.001732

Nonparametric



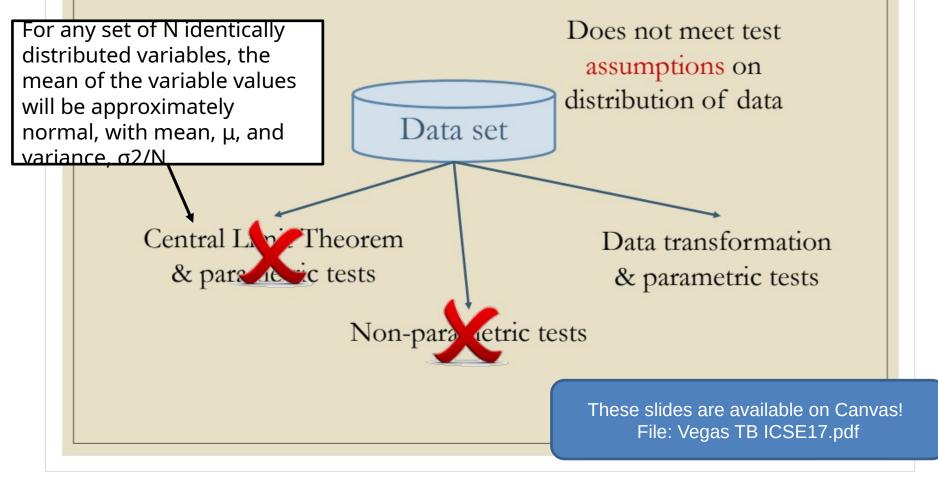
Main statistical tests

You are mea subjects	suring different su	u are measuring the same bject against different eatments	Use this in case the values of your dep. var are not normally distributed
Outcome Variable	Are the observations independent	correlated	Alternatives if the normality assumption is violated (and small sample size):
Continuous (e.g. pain scale, cognitive	Ttest: compares means between two independent groups ANOVA: compares means between more than two	Paired ttest: compares means between two related groups (e.g., the same subjects before and after)	Non-parametric statistics Wilcoxon sign-rank test: non-parametric alternative to the paired ttest
function)	Pearson's correlation coefficient (linear correlation): shows linear correlation between two	Repeated-measures ANOVA: compares changes over time in the means of two or more groups (repeated measurements)	Wilcoxon sum-rank test (=Mann-Whitney U test): non- parametric alternative to the ttest Kruskal-Wallis test: non- parametric alternative to ANOVA
	continuous variables Linear regression: multivariate regression technique used when the outcome is continuous; gives slopes	Mixed models/GEE modeling: multivariate regression techniques to compare changes over time between two or more groups; gives rate of change over time	Spearman rank correlation coefficient: non-parametric alternative to Pearson's correlation coefficient

Data transformation







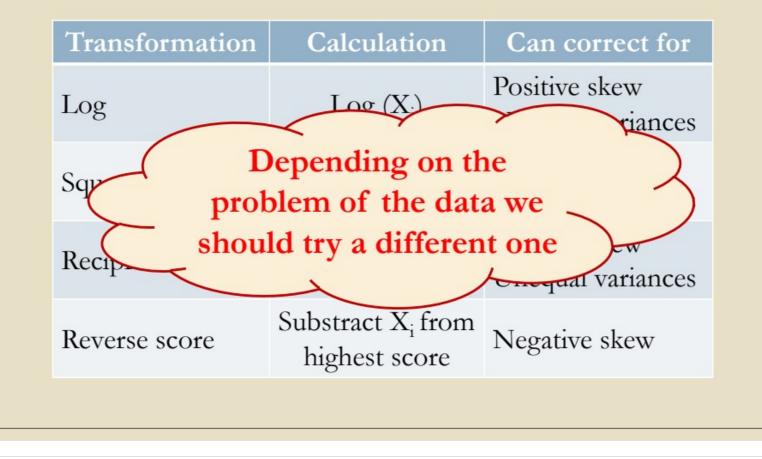
DATA TRANSFORMATION

- ° Corrects several problems in data:
 - Non-normality
 - Unequal variances
- ° Will not change the relationships between variables
 - ° The relative differences between scores for a given variable stay the same
- Does change the differences between different variables
 - ° Changes units of measurement

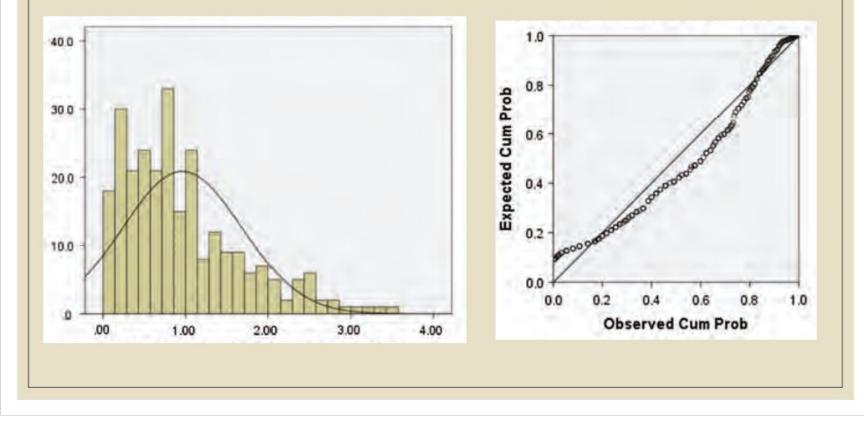
(SOME) POSSIBLE TRANSFORMATIONS

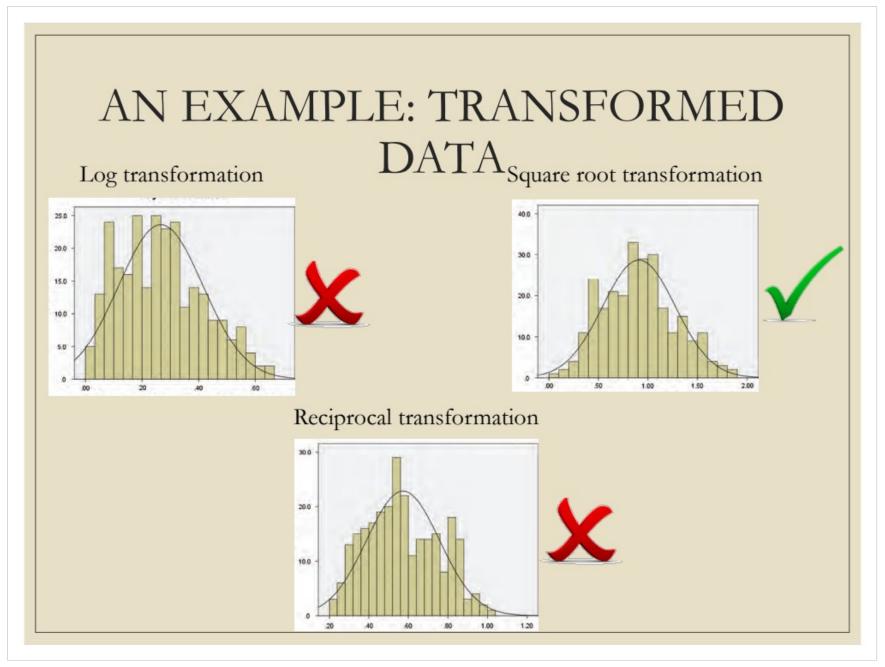
Transformation	Calculation	Can correct for
Log	Log (X _i)	Positive skew Unequal variances
Square root	$\sqrt{X_i}$	Positive skew Unequal variances
Reciprocal	$1/X_i$	Positive skew Unequal variances
Reverse score	Substract X _i from highest score	Negative skew

(SOME) POSSIBLE TRANSFORMATIONS



AN EXAMPLE: ORIGINAL DATA





DISCUSSION

° Data transformation looks like a better option

° Do not forget to "un-transform" for data interpretation

- ° Un-transformation is necessary for:
 - Mean
 - ° Confidence interval for mean
- ° Un-transformation is not necessary for:
 - ° Significance
 - Power

TIPS

- When selecting statistical test check test assumptions on distribution of data
- If distribution assumptions are not met:
 Do not use limit central theorem
 - ° Try data transformation first
 - ° If it does not work, use non-parametric tests
- ° If you transform, do not forget un-transform



- You can use the <u>bestNormalize</u> package to discover the best transformation to apply
- Remember to apply the same transformation to ALL the measures of a dependent variable

multiple variables, in case you analyze interactions

- When you will visualize tables and plots you will need to show the non-transformed data
- If you do not manage to satisfy the assumptions of your statistical test (after transforming), then indeed you can go with a non-parametric one (this is always the safest way, even though it might not be very powerful)



Correction of p-values



Example

Dependent variable = energy consumption of the app Independent variables =

- A: Image encoding algorithm: {png, jpeg}
- B: Mobile device type: {high-end, low-end}
- C: Network conditions: {wifi, 3G}

You perform 3 tests:

- t.test(A, B)
- t.test(A, C)
- t.test(B, C)

P(at least one significant result) = 1 - P(no significant results) = 1 - $(1 - 0.05)^3$ ≈ 0.15

 \rightarrow 15% chance of seeing relevant results, when there may be none



The problem

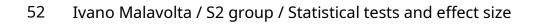
Multiple tests (statistically \rightarrow higher probability of getting

significant) results

 \rightarrow you have to adjust your α (it was 0.05)

Three main correction techniques:

- Bonferroni
- Holm
- Benjamini- Hochberg





Bonferroni correction

Supposing we are doing N tests,

we can reject H_0 if the p-values of those tests are below α/N

We can reject the H_0 if a test provides a p-value < 0.05/3=0.016

 \rightarrow 0.016 is our new significance threshold!

```
> p.adjust(c(0.01,0.02,0.03),method="bonferroni")
[1] 0.03 0.06 0.09
```

```
Usage
p.adjust(p, method = p.adjust.methods, n = length(p))
p.adjust.methods
# c("holm", "hochberg", "hommel", "bonferroni", "BH", "BY",
# "fdr", "none")
```



Less stringent that Bonferroni's

Procedure:

- rank your p-values from the smallest to the largest
- multiply the first by N, the second by N-1, etc.
- a p-value is significant if, after multiplied, it is <0.05

P-values of the tests: {0.01, 0.02, 0.03}





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Benjamini- Hochberg 's correction

The least stringent correction (**highly suggested**)

Procedure:

- rank your p-values from the smallest to the largest
- assign ranks to each p-value according to its position
 - first=1, second=2, third=3, ...
- compute the BH critical value for each p-value as (i/N)Q
 - i = the ith p-value
 - N = the total number of p-values
 - Q = the acceptable false discovery rate as percentage (e.g., 50%)
- identify P as the highest p-value that is smaller than the BH critical value

e considered as signification

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Benjamini- Hochberg 's correction

P-values of the tests: {0.01, 0.02, 0.03, 0.04, 0.2, 0.4, 0.8, 0.9}

Original p- value	Rank	BH
0.01	1	(1/8)*0.5= 0.0625
0.02	2	(2/8)*0.5= 0.125
0.03	3	(3/8)*0.5= 0.1875
0.04	4	(4/8)*0.5= 0.25
0.2	5	(5/8)*0.5= 0.3125
0.4	6	(6/8)*0.5= 0.375
0.8	7	(7/8)*0.5= 0.4375
0.9	8	(8/8)*0.5= 0.5



Effect size









Effect Size: quantitative measure of the **strength** of a phenomenon

Actual difference:
 0.0001%







Effect size measures

Cohen's d

parametric statistics \bigcirc Cliff's delta non-parametric statistics Parametric Non-parametric Design Chi-2, Binomial test One factor, one treatment t-test, F-test Mann-Whitney, Chi-2 One factor, two treatments, completely randomized design Paired t-test One factor, two treatments, paired comparison Wilcoxon, Sign test ANOVA Kruskal-Wallis, Chi-2 One factor, more than two treatments More than one factor ANOVA^a

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Cohen's d

The magnitude of a main factor treatment effect on the dependent variable

> $d = \frac{\bar{x_1} - \bar{x_2}}{s}$ 0 = full overlap 1 = 1-sigma distance between the means 3 = 3-sigma distance → ~no overlap

Values:

Where:

• x_1 , x_2 = the means of the two groups

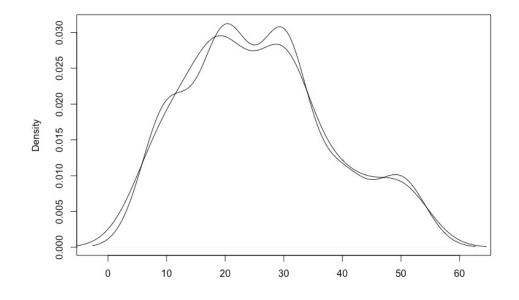
s = standard deviation			
negligible	small	medium	large
0.	.2 0.	.5 0.	.8



Cohen's d in R

- > treatment1 <- c(10,10,20,20,20,30,30,30,40,50)
- > treatment2 <- c(12,8,20,20,18,30,30,30,40,50)
- > cohen.d(treatment1,treatment2, paired=F, pooled=F)

Cohen's d





Cliff's delta

Represents the degree of overlap between the two distributions of scores

$$d = rac{\#(x_i > x_j) - \#(x_i < x_j)}{}$$

mn

Values:

+1 = all the values of one group > all the values of

0 = full overlap

the other one

-1 = the inverse

Where:

- $x_i =$ the values of the first group
- $x_i =$ the values of the second group
- m, n = the cardinalities of the two groups

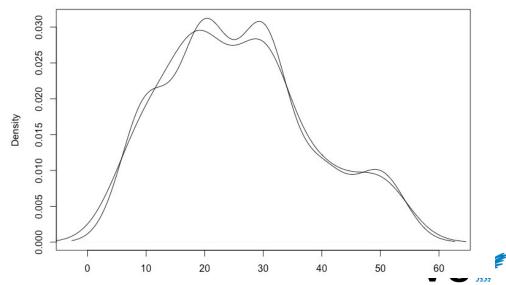
		negligible	small	medium	large	4
62	Ivanc	0.	.147 (0.33 0.	.47	VU

Cliff's delta in R

- > treatment1 <- c(10,10,20,20,20,30,30,30,40,50)</pre>
- > treatment2 <- c(12,8,20,20,18,30,30,30,40,50)</pre>
- > cliff.delta(treatment1,treatment2)

Cliff's Delta

delta estimate: 0.03 (negligible) 95 percent confidence interval: inf sup -0.4603148 0.5062902



What this lecture means to you?

Tasks for data analysis

- 1. Descriptive statistics
- for understanding the "shape" of collected data
- 2. Select statistical test • according to collected metrics and data distribution
- 3. Hypothesis testing
 - for providing evidence about your findings
 - i. statistical significance

- for understanding if your (statistically significant) results are actually relevant 4. Effect size calculation
 - in practice
- 5. Power analysis
 - o for knowing if your results :

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DATA: BY THE NUMBERS



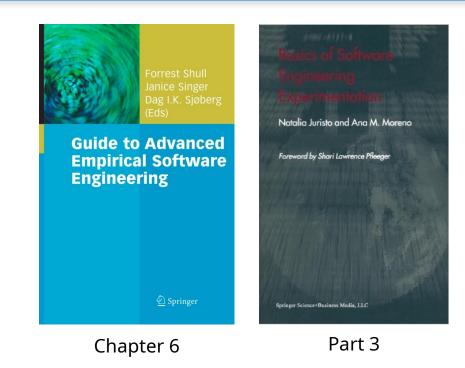






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Readings



Slides of Sira Vegas's technical briefings at ICSE 2017 (on Canvas)

[1] Dybå, Tore, Vigdis By Kampenes, and Dag IK Sjøberg. "A systematic review of statistical power in software engineering experiments." Information and Software Technology 48.8 (2006): 745-755.

VU



Some contents of lecture extracted from:

• Giuseppe Procaccianti's lectures at VU

